

# Gauge Parameter Dependence in Gauge Theories

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## Abstract

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On the example of topologically massive gauge field theory we find the origin of possible inconsistency of working with gauge fixing terms (together with relevant ghost sector).

The common procedure of practical calculations in e.g. QED and QCD is the introduction of gauge-fixing terms into Lagrangian together with the relevant ghost sector. It is understood that these terms should not influence physical quantities. However, encountering gauge parameter dependence of some quantity it is not easy to understand whether it is the gauge fixing procedure that is inconsistent or it is the fault of the theory itself. Below we are going to show that possible source of inconsistency of working with gauge fixing terms resides in averaging the generating functional for Green's functions for different gauges. Accordingly, when the mentioned procedure of averaging is inconsistent, the quantization in Hamilton formalism is not equivalent to the Lagrange formalism quantization (with the help of introduction of gauge fixing terms).

The quantization rules for gauge theories in Lagrange formalism were derived in [1]. It is well known that this rules must be modified for more general theories [2]. The main method to construct a covariant gauge quantum theory is the BRST quantization [3]. Equivalence of the Lagrangian and Hamiltonian approaches for the BRST quantization was demonstrated in [4].

For an illustration we are going to use an example where the Faddeev-Popov quantization in the Hamilton formalism produces correct results, while quantization in the Lagrange formalism by adding covariant gauge fixing term leads to the gauge parameter dependence of physical quantities.

Consider topologically massive abelian gauge field coupled to fermion (in 2+1 dimensions) [5]:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{M}{4}\epsilon_{\mu\nu\lambda}A^\mu F^{\nu\lambda} + \bar{\psi}(i\hat{\partial} + e\hat{A} - m)\psi. \quad (1)$$

In [5] it was noticed that the fermion pole mass calculated in the covariant gauge (i.e.  $L_{gf} = (1/2\xi)(\partial A)^2$ ) at one loop depends on the gauge fixing parameter  $\xi$ . The wave function renormalization constant as well as the S-matrix elements are infrared (IR) divergent in any covariant gauge other than the Landau one. Authors of [5] found problematic to define the mass-shell in this theory in gauge invariant way. We are going to show that in fact these gauge parameter dependence problems are due to the inconsistency of the gauge fixing procedure.

We have checked by explicit calculations that the fermion pole mass in the Coulomb and axial gauges equals to the Landau gauge ( $\xi = 0$ ) one. (In [5] it was noted that the IR safe version of the gauge field propagator in the Coulomb gauge produces the Landau gauge pole mass. We just checked that the same is true for the full propagator.) Besides, the S-matrix elements, being finite in the Landau (and also in the Coulomb and axial gauges), suffer from severe IR divergences in any other covariant ( $\xi \neq 0$ ) gauge.

To investigate the origin of this apparent puzzle we have performed the canonical quantization of this theory in the physical (gauge invariant) variables and found that the resulting quantum theory coincides with canonically quantized theory in Coulomb gauge. We do not reproduce quantization in physical variables here — it is analogous to the ordinary QED case [6].

So, for this theory, the Coulomb gauge is directly related to the quantization in gauge invariant variables and hence any other consistent quantization scheme must reproduce the Coulomb gauge results.

In the Lorentz invariant gauge  $\partial_\mu A^\mu = C(x)$  the generating functional for the Green's functions is given in the form [7]:

$$Z_C[J, \zeta, \bar{\zeta}] = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \delta(\partial_\mu A^\mu - C(x)) \det \mathcal{M}_L e^{iS + i \int dx (J^\mu A_\mu + \bar{\psi} \zeta + \bar{\zeta} \psi)}. \quad (2)$$

It is easy to see that for the choice  $C(x) = 0$  the Green's functions generated by (2) are identical to the Landau gauge ones. So both Landau and Coulomb gauges are implied from Hamilton formalism quantization. Note that in our example problems start in the covariant non-Landau gauges.

Now let us recall how the Lagrange formalism quantization by adding the gauge fixing terms to the Lagrangian can be reproduced from (2). Multiply (2) by  $\exp\{\frac{i}{2\xi} \int C^2(x) dx\}$  and integrate over  $C(x)$ . We get:

$$Z[J, \zeta, \bar{\zeta}] = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \det \mathcal{M}_L e^{i \int dx \{L + \frac{1}{2\xi} (\partial A)^2 + J^\mu A_\mu + \bar{\psi} \zeta + \bar{\zeta} \psi\}}. \quad (3)$$

Eq. (3) is an expression for the generating functional which is usually used in the framework of the Lagrange quantization. We gave this formal derivation of (3) because it clearly indicates that the addition of gauge fixing terms amounts to the averaging of different gauges  $\partial_\mu A^\mu = C(x)$  with the weight  $\exp\{\frac{i}{2\xi} \int C^2(x)dx\}$ . In fact it does not correspond to any definite gauge at all [8], rather it is a mixture of different gauges.

For any definite  $C(x)$ ,  $Z_C$  leads to the  $C(x)$ -independent physical quantities. Usually it is conjectured that averaging  $Z_C$  for different  $C(x)$  will translate this  $C(x)$  independence into the gauge parameter —  $\xi$  independence. We claim that this averaging procedure may ruin equivalence of (2) and (3). The situation is similar to the infinite sum of vanishing terms — the sum may turn out to be finite or even divergent.

Indeed let us examine the  $C(x)$  dependent gauge field propagator in the model (1):

$$D_{\mu\nu}^C = \frac{1}{N} \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \delta(\partial_\mu A^\mu - C(x)) A_\mu A_\nu e^{iS}. \quad (4)$$

Let us perform gauge transformation  $A_\mu \rightarrow A_\mu + \partial_\mu \partial^{-2} C(x)$ :

$$D_{\mu\nu}^C(p, q) = \delta(p - q) D_{\mu\nu}^L(p) + C(p) C(q) \frac{p_\mu q_\nu}{p^2 q^2}.$$

Here  $D^L$  denotes propagator in the Landau gauge and we have passed to the momentum space. (The function  $C(x)$  breaks translational invariance and results in the nondiagonal term.) Now, if we define the fermion mass and wave function renormalization constant from the diagonal part, it is easy to see that no IR problems will arise while calculating the physical amplitudes. (Although it seems trivial from simple power counting, we explicitly checked it for fermion scattering in the external field.) If we integrate (4) over  $C$  with the weight  $\exp\{\frac{i}{2\xi} \int C^2(x)dx\}$  we will get the usual propagator corresponding to the gauge fixing Lagrangian  $(1/2\xi)(\partial_\mu A^\mu)^2$ :

$$D_{\mu\nu} = \frac{-i}{p^2 - M^2} (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - \frac{iM}{p^2} \epsilon_{\mu\nu\lambda} p^\lambda) + \xi \frac{p_\mu p_\nu}{p^4}. \quad (5)$$

Note the singular behaviour of the  $\xi$ -dependent part. It causes gauge parameter dependence of the fermion pole mass and also IR divergences of the

$\mathcal{S}$ -matrix elements. Contribution of this term to the fermion pole mass is proportional to:

$$\delta m_\xi \sim (p^2 - m^2)^2 \int d^n q \frac{1}{q^4 ((p - q)^2 - m^2)} \Big|_{p^2 \rightarrow m^2, n \rightarrow 3}. \quad (6)$$

Due to the infrared divergence of the integral this expression will yield finite gauge parameter dependent contribution to the mass. In the same manner there arise IR divergences in the S-matrix elements, which are absent in the physical Coulomb, axial and Landau gauges. Obviously in the described situation the LSZ formalism does not commute with the integration over  $C$ . Indeed, if we calculate Green's functions in  $\partial_\mu A^\mu(x) = C(x)$  gauge and calculate physical quantities using LSZ formalism, we find that they are well defined and do not depend on  $C(x)$ . Integration over  $C(x)$  leads only to the numerical factor which in fact is cancelled by the normalization factor. On the other hand, integrating  $Z_C[J, \zeta, \bar{\zeta}]$  over  $C(x)$  and applying LSZ formulas we find that physical quantities are IR divergent and  $\xi$ -dependent. So, LSZ formalism and integration over  $C(x)$  do not commute in this particular case (and hence, in general).

The generating functional (3) may be derived also with the help of BRST quantization. So it provides us with one more example [9] when the BRST quantization fails.

This kind of effect may happen in four dimensions too if the averaging weight function is chosen in the form [8]:

$$\exp\left\{\frac{i}{2\xi} \int [f(\partial^2)C]^2(x) dx\right\} \quad (7)$$

where  $f$  is an arbitrary function of D'Alembert operator. Choosing  $f = (\partial^2)^k$  will lead to  $(q^2)^{-(k+1)}$  singularity in the gauge field propagator and hence result additional (incurable without some artificial *ad hoc* procedure) infrared problems in ordinary QED and QCD.

Of course one may try to foresee in what circumstances will the described problem arise. Consider again the covariant gauge. In the propagator of gauge particle the tensor structure of the gauge fixing term  $(p_\mu p_\nu)$  is present also in the Landau gauge propagator. Evidently problems begin if the IR behaviour of the coefficient of this tensor structure in gauge fixing part is more singular then the same in the Landau gauge. It is clear enough because, normally, due to the gauge invariance, such structure will not affect

physical quantities. Sometimes comparison of IR behaviour of terms of the bare propagator may be misleading. E.g., at the first sight, the above criterion is not satisfied for conventional QED in three dimensions. However, in this theory the Chern-Simons term is generated dynamically [5] (discussion of regularization dependence of this fact noted in [5], can be found in [10]) changing the IR behaviour of the Landau gauge terms. Similar analysis must be applied to the other gauge fixing conditions together with weight functions in order find when does the averaging procedure introduce additional higher order IR singularities.

So we have demonstrated that at certain cases Lagrange and Hamilton formalism quantizations are not equivalent due to inadmissibility of averaging different gauge fixing conditions. Hence tests of gauge parameter independence of the physical quantities not always serve for checking consistency of the theory, but rather represent a test for applicability of Lagrange quantization with gauge fixing Lagrangian and of BRST quantization. The possible source of inconsistency of calculations with gauge fixing Lagrangeans resides in the seemingly ‘innocent’ procedure — averaging different gauges with some weight (which is equivalent to addition of the gauge fixing terms).

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